# Lecture 9

## The Warehouse Logistic

The industry has found a very important issue in warehousing. It is a key part of the supply chain management and focuses on controlling the movement and storage of materials within a warehouse and processing the associated transactions, including shipping, receiving, and picking.

# Warehouse Management

- Strategic level: warehouse layout design;
- Tactical level: storage policy (where each product should be located), for example, demand−based storage, class−based storage, …
- Operational level: order picking, i.e. the process of retrieving items from their storage locations in response to a specific customer request.

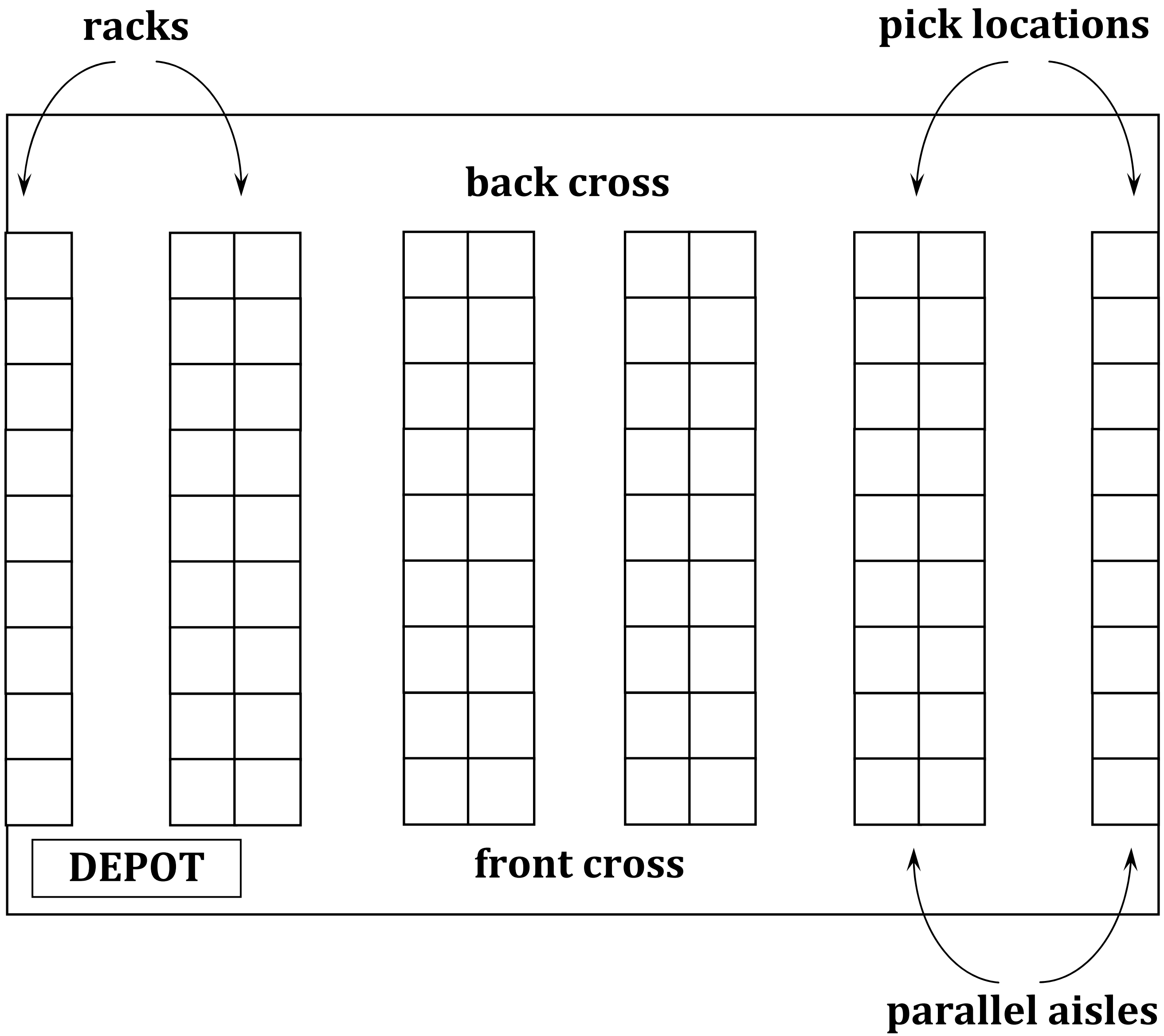
# Order Picking

The order picking operations are the most important processes in a warehouse. The cost of this process may constitute more than 60% of the total operating cost. A warehouse receives every day several orders from its customers.

Each ***order*** consists of a list of some ***items*** that have to be retrieved from the warehouse and shipped to a specific customer.

Items must be collected by an ***operator***. Several orders are put together into ***batches***, satisfying a fixed capacity constraint. Then, each batch is assigned to an operator, who retrieves all the items included in those orders grouped into corresponding batch in a single tour.

# Warehouses layout with rectangular shape and parallel ailes



# Example of a set of orders

| 01 | |
|---|---|
| *Apples* |  |
| *Cherries* |  |
| *Carrots* |  |

*Weight* = 3

| 02 | |
|---|---|
| *Bananas* |  |
| *Garlics* |  |

*Weight* = 2

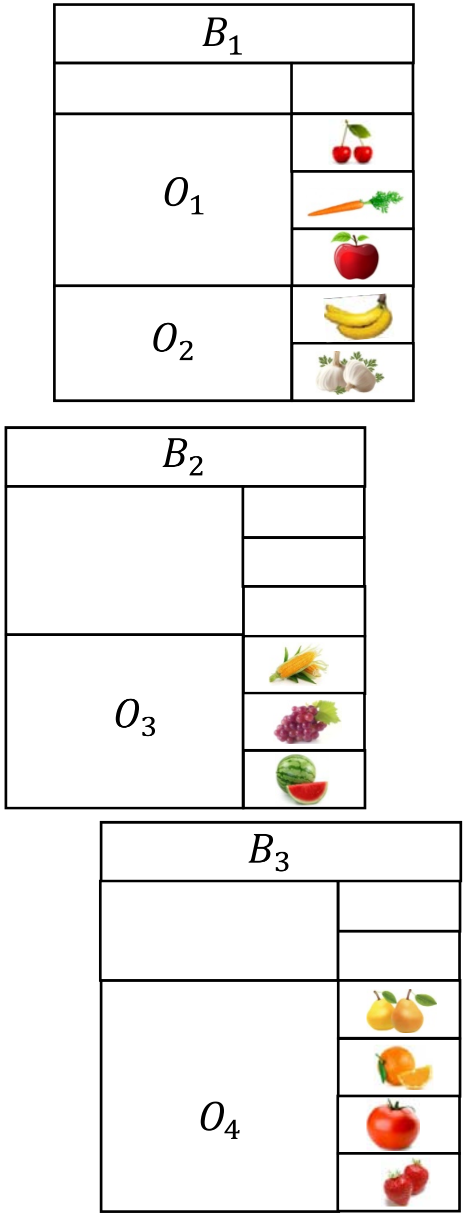
| 03 | |
|---|---|
| *Corns* |  |
| *Grapes* |  |
| *Watermelon* |  |

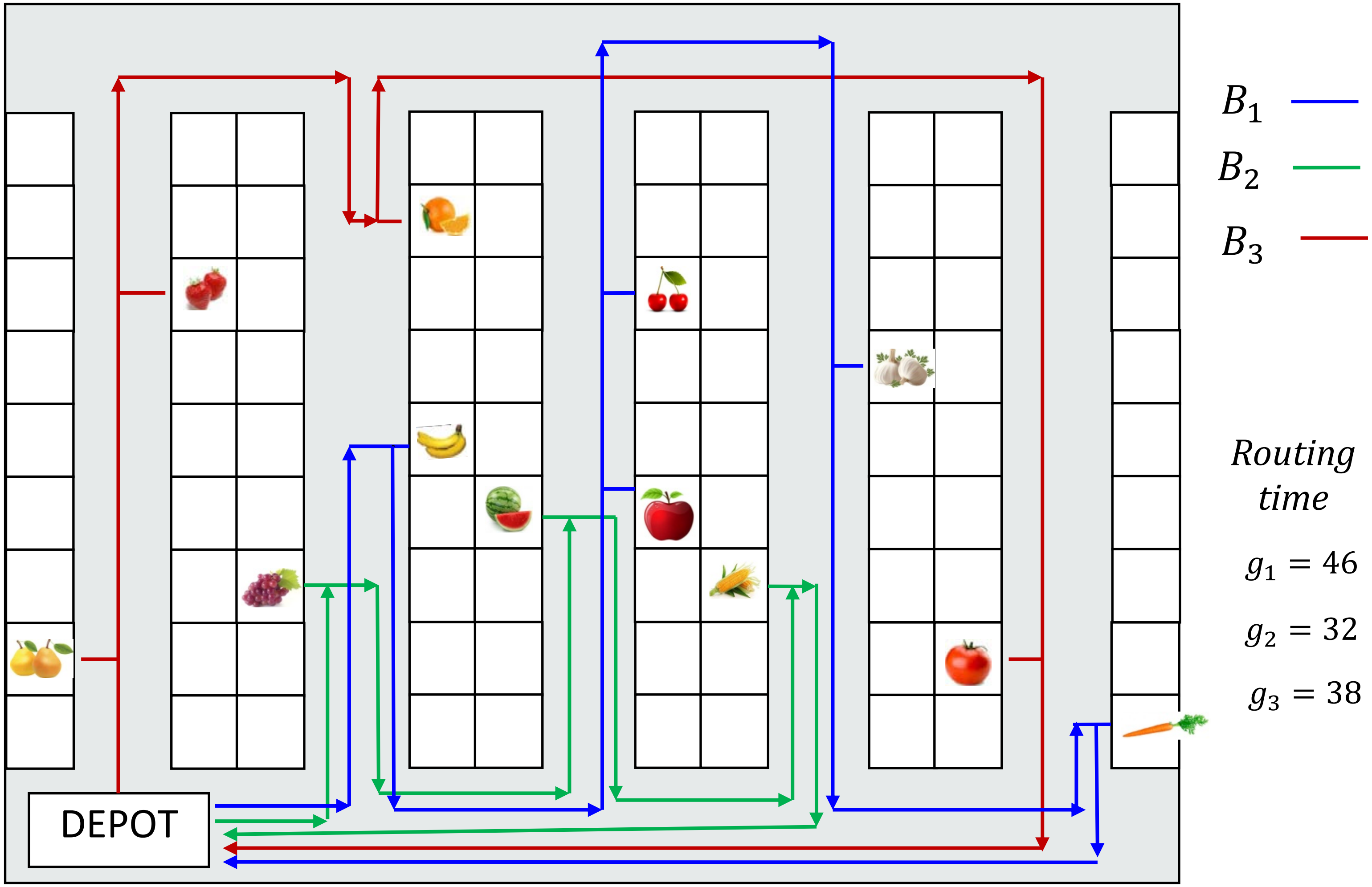
*Weight* = 3

| 04 | |
|---|---|
| *Pears* |  |
| *Oranges* |  |
| *Tomatoes* |  |
| *Strawberries* |  |

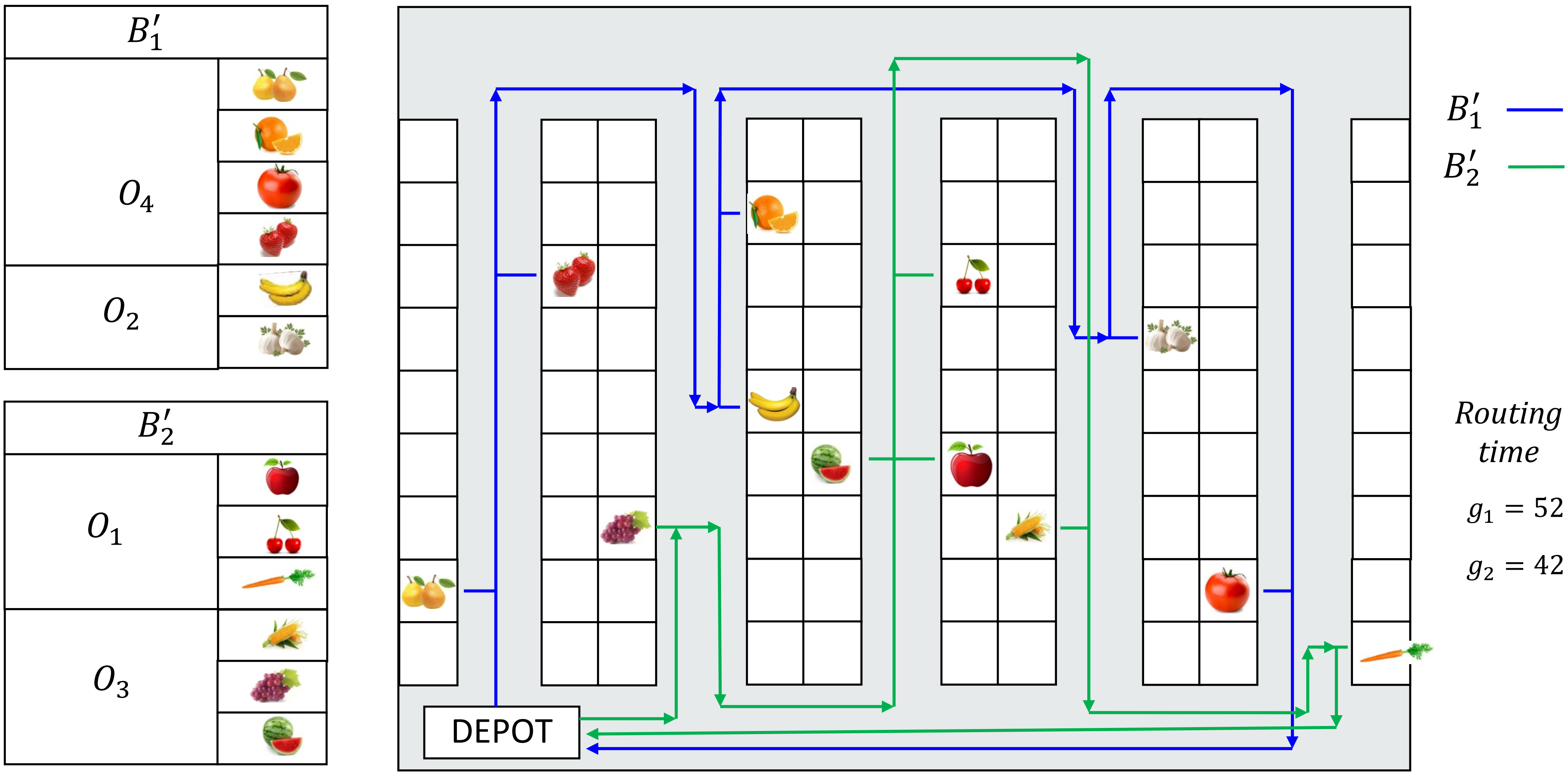
*Weight* = 4

# Example of a set of batches from the orders



# Example of a set of batches from the orders



# The Order Batching Problem

**Given**

$I$ is the set of orders, $|I| = n$;

$J$ is the set of batches, $|J| = n$;

$K$ is the set of aisles;

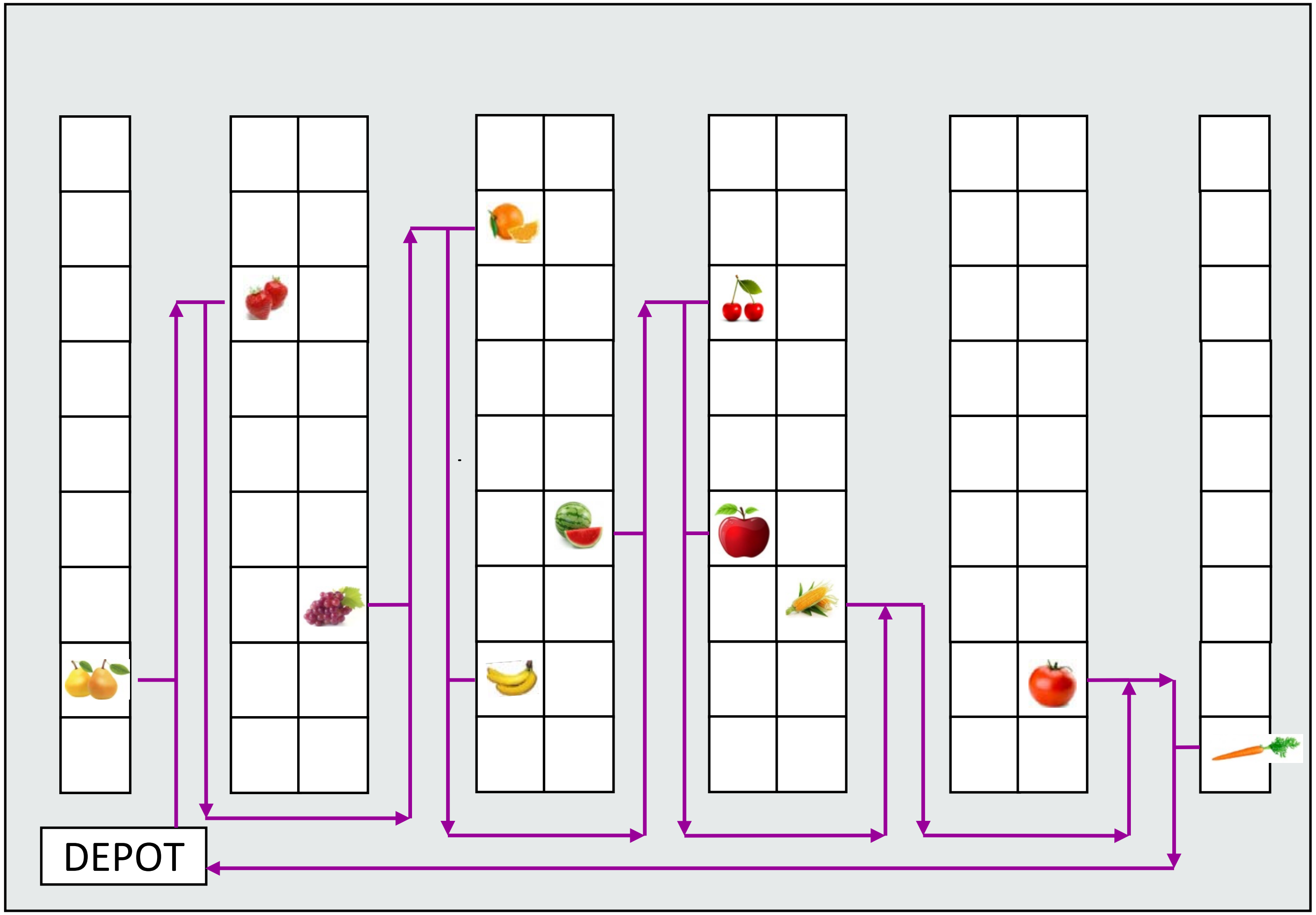
$Q$ is the capacity of batch (a cart of operator);

$m_i$ is the number of items in order $i$;

$d_{ik}$ is the maximal vertical distance that the operator serving order $i$ should travel along aisle $k$ starting from the front aisle (may be 0 if we skip aisle $k$).

**Find** the best collection of the batches to minimize the total travel distance to retrieve all items.

# Return policy



# Mathematical Model

**Variables:**

$$x_{ij} = \begin{cases} 1 & \text{if order } i \text{ is assinged into batch } j \\ 0 & \text{otherwise} \end{cases}$$

$$z_j = \begin{cases} 1 & \text{if batch } j \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

The batch packing subproblem:

$$(*) \quad \begin{cases} \sum_{j=1}^{n} x_{ij} = 1, \quad i \in I; \\ x_{ij} \leq z_j, \quad j \in J, i \in I; \\ \sum_{i=1}^{n} m_i x_{ij} \leq Q, \quad j \in J. \end{cases}$$

*If we wish to minimize the number of batches, what kind of objective function we need?*

## Additional variables

$$y_{jk} = \begin{cases} 1 & \text{if operator serving batch } j \text{ must visit aisle } k \\ 0 & \text{otherwise;} \end{cases}$$

$h_j \geq 0$ stands for the one way horizontal distance traveled by operator serving batch $j$ starting from $I/0$ point;

$s_{jk} \geq 0$ indicates one way maximal vertical distance traveled in aisle $k$ starting from the front aisle to the location of an item which belongs to an order assigned into batch $j$.

## Model for Return Policy

$$\min\left(2\sum_{j\in J} h_j + 2\sum_{j\in J}\sum_{k\in K} s_{jk}\right)$$

s.t. $(*)$ and

$$d_{ik}x_{ij} \le s_{jk},\ \ i \in I, j \in J, k \in K;$$

$$kw\, y_{jk} \le h_j,\ \ j \in J, k \in K;$$

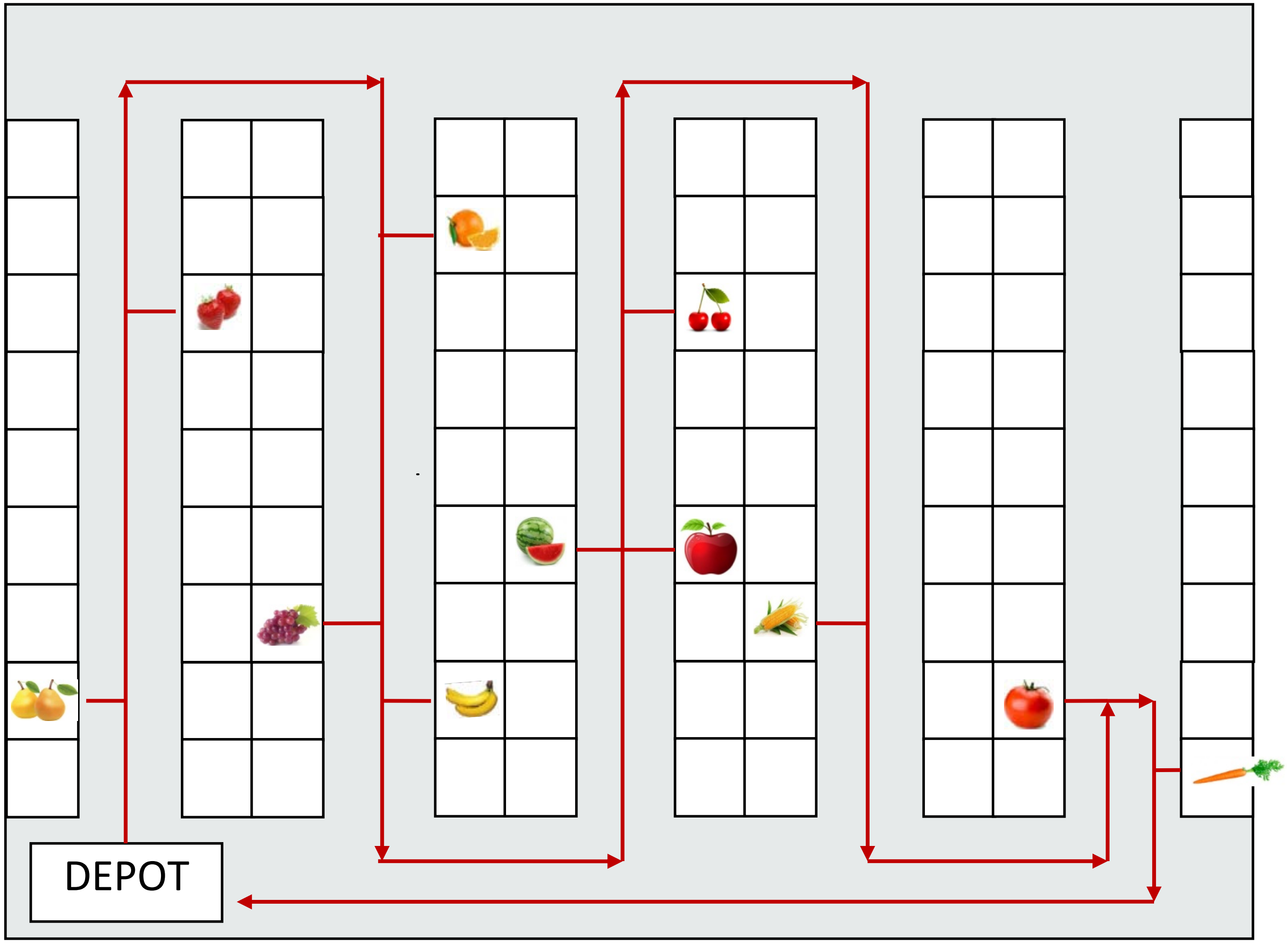
$$\sum_{i\in I} d_{ik}x_{ij} \le My_{jk},\ \ j \in J, k \in K;$$

$$y_{jk}, x_{ij} \in \{0,1\},\ \ s_{jk} \ge 0,\ \ h_j \ge 0,\ \ i \in I, j \in J, k \in K.$$

Reformulate the model for the case when

- $I/0$ is located in front of the aisle number $k^*$;
- we must pay salary for the operators;
- number of orders in each batch must be in interval $[b^1, b^2]$;
- weight of each batch cannot exceed the upper bound $\Omega$;
- we have $m$ operators and wish to minimize the total time for all orders when operators work at the same time in parallel.

# Traversal Policy



## Traversal Policy

$$x_{ij} = \begin{cases} 1 & \text{if order } i \text{ is assinged into batch } j \\ 0 & \text{otherwise} \end{cases}$$

$$z_j = \begin{cases} 1 & \text{if batch } j \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{jk} = \begin{cases} 1 & \text{if operator serving batch } j \text{ must visit aisle } k \\ 0 & \text{otherwise;} \end{cases}$$

$v_j \geq 0$, integer, denotes the number of times the operator serving batch $j$ performs two-way traversals;

$$c_j = \begin{cases} 1 & \text{if number of aisles is odd for batch } j \\ 0 & \text{otherwise;} \end{cases}$$

$$p_{jk} = \begin{cases} 1 & \text{if aisles } k \text{ is the rightmost ailes for batch } j \\ 0 & \text{otherwise} \end{cases}$$

$u_{jk} \geq 0$ indicates the vertical one way distance traveled in the rightmost aisle $k$ by the operator serving batch $j$ and visiting totally an odd number of aisles.

# Model for Traversal Policy

$$\min\left(2\sum_{j\in J}\sum_{k\in K}u_{jk} + 2\sum_{j\in J}h_j + 2L\sum_{j\in J}(v_j - c_j)\right)$$

s.t. $(*)$ and

$$kw\, y_{jk} \le h_j,\quad j\in J, k\in K;$$

$$\sum_{i\in I} d_{ik}x_{ij} \le My_{jk},\quad j\in J, k\in K;$$

$$\sum_{k\in K} y_{jk} + c_j = 2v_j,\quad j\in J;$$

$$d_{ik}x_{ij} \le u_{jk} + M(1 - p_{jk}) + M(1 - c_j),\quad i\in I, j\in J, k\in K;$$

$$y_{jk} - \sum_{l\ge k+1} y_{jl} \le p_{jk} \le y_{jk},\quad j\in J, k\in K;$$

$v_j \ge 0$, integer, $x_{ij}, y_{jk}, p_{jk}, c_j \in \{0,1\}, u_{jk} \ge 0, h_j \ge 0.$

## General Case (The Best Policy)

$I$ is the set of orders;

$J$ is the set of *all* feasible batches;

$m_i$ is the capacity utilization required by order $i$;

$Q$ is the capacity of picking device;

$P_j$ is the length of a picking tour in which all orders of batch $j$ are collected;

$a_{ij} \in \{0,1\}$ indicate orders in each batch.

We assume that

$$\sum_{i \in I} a_{ij} m_i \leq Q, \quad j \in J.$$

## Mathematical Model

$$\min \sum_{j \in J} P_j z_j$$

s.t.

$$\sum_{j \in J} a_{ij} z_j = 1, \quad i \in I;$$

$$z_j \in \{0,1\}, \quad j \in J.$$

*What is the weakest side of the model?*

**Hometask.** Design an exact polynomial time algorithm for the case when each batch contains at most two orders.

## Order Batching and Batch Scheduling

In order picking systems, the customer orders have to be completed and provided by certain ***due dates***. In distribution warehouses due dates have to be met in order to guarantee the schedule departure of trucks.

In addition to not allowing orders to be late, it also not acceptable that the items are provided a long time ahead of the due date, since that would result in an unnecessary accumulation of material or work-in-progress.

In such cases, instead of measuring the quality of a solution by means of the total picking time or the total length of the tours, the batching will have to be evaluated with respect to both ***earliness*** and ***tardiness*** of the orders.

# Scheduling Problem for an Operator

**Given:** $d_i$ is the due date of order $i \in I$;

Variables:

$c_j \geq 0$ is the completion time of batch $j \in J$;

$p_j > 0$ is processing time for batch $j \in J$;

$t_{ij} \geq 0$ is tardiness of order $i$ in batch $j$ (if $a_{ij} = 1$);

$e_{ij} \geq 0$ is earliness of order $i$ in batch $j$ (if $a_{ij} = 1$);

$v_{jk} \in \{0,1\}$ is 1 iff batch $j$ is released before batch $k$.

**Goal:** We wish to schedule the batches and minimize the weighted sum of the total earliness and the total tardiness of all customer orders with respect to due dates.

## Mathematical Model

$$\min \sum_{i \in I} \sum_{j \in J} a_{ij} t_{ij}$$

s.t.

$$\sum_{j \in J} a_{ij} z_j = 1, \;\; i \in I;$$

$$p_j z_j \le c_j, \;\; j \in J;$$

$$t_{ij} \ge c_j - d_i z_j, \;\; i\text{:}\;\; a_{ij} = 1, \;\; j \in J;$$

$$t_{ij} \ge 0, \;\;\; i \in I, j \in J;$$

$$c_j + M v_{jk} \ge c_k + p_j z_j;$$

$$c_k + M\left(1 - v_{jk}\right) \ge c_j + p_k z_k;$$

$$v_{jk}, \;\; z_j \;\; \in \{0,1\}, j \in J, k \in K.$$

*How to modify the model by including the earliness of the orders?*