Is there a $(4, 27, 2)$ partial geometry?

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A *partial geometry* with parameters (s, t, α) consists of lines and points with the properties that (i) each line has $s + 1$ points and two distinct lines intersect in at most one point; (ii) each point is on $t + 1$ lines and two distinct point occur on at most one line; and (iii) for each point p that does not lie on a line l, there are exactly α lines through p that intersect l. The question whether there exists a $(4, 27, 2)$ partial geometry has tantalized researchers during the last couple of decades. Such a partial geometry would have 275 points and 1540 lines and its point graph would be a $(275, 112, 30, 56)$ strongly regular graph (srg). There is a unique srg with the aforementioned parameters called the *McLaughlin graph*. In this talk, a computer search for a $(4, 27, 2)$ partial geometry starting from the McLaughlin graph is described. After 270 core-years and more than one physical year, the computers claim that there is no such partial geometry.