text, a more natural structure is a λ-clone with infinitary, $(1+\bar{i})$-ry, applications and infinitary, $\bar{i}$-ry, λ-quantifiers for all, finite and countable, sequences $\bar{i} \in \omega^{\infty}$. In this way we obtain an algebraic version of an infinitary λ-calculus but this is another story.

REFERENCES

1. MEYER A.R. What is a model of the lambda calculus? Information and Control, 52,1, 1982(87-122).

2. DISKIN Z.B. Lambda term systems. Z.Math.Logic Grundl. Math. Submitted.

ON CODING OF HEREDITARILY-FINITE SETS, POLYNOMIAL-TIME COMPUTABILITY AND Δ-EXPRESSIBILITY

Sazonov V.Yu., Leontjev A.V., Pereslavl-Zalessky

This paper is devoted to computability and definability in terms of bounded (i.e.,Δ-) set theoretic language (cf. references below).

A coding (or numbering; cf. the general theory in [3]) of the universe of hereditarily-finite sets HF is any surjection $\vartheta : A^* \to HF$ from the set of all finite strings over some finite alphabet A. Let P_ϑ denote the class of operations F: HF → HF such that $F\vartheta = \vartheta f$ for some polynomial-time computable (or shortly, P-) function $f: A^* \to A^*$. For any two codings $\vartheta : A^* \to HF$, $\eta: B^* \to HF$ and P-function $f: A^* \to B^*$ the P-reducibility $\vartheta = \eta f$ is denoted also as $\vartheta \leq^f_P \eta$ or $\vartheta \leq_P \eta$. P-equivalence $\vartheta \equiv_P \eta$ means $\vartheta \leq_P \eta$ & $\eta \leq_P \vartheta$ and implies $P_\vartheta = P_\eta$. If cardinalities of A and B are ≥ 2 then any $\vartheta : A^* \to HF$ is P-equivalent to some $\vartheta: B^* \to HF$ (via arbitrary two-sided P-bijection $f: A^* \to B^*$). Hence, we will usually consider codings over the same A. Any ϑ is called P-coding if (1) the predicate "$HF \models \vartheta(a) \in \vartheta(b)$" is P-decidable on any, $a,b \in A^*$ and (2) two P-computable mappings $a \mapsto a_1,\ldots,a_k$ and $a_1,\ldots$

$\ldots,a_k \mapsto a$ from codes to lists of cods and conversely exist such that in both cases $\{\vartheta(a_1),\ldots,\vartheta(a_k)\} = \vartheta(a)$ in HF. Alternatively, P^*-coding is one satisfying (1), as above, and, in place of (2), the condition (2^*) on P-computability of a mapping $a \mapsto a_1,\ldots,a_k$ such that $\{\vartheta(a_1),\ldots,\vartheta(a_k)\}$ = the transitive closure of set $\vartheta(a)$ in HF.

Examples of P & P^*-codings are 1) (correct) bracket expressions, like {}, {{}{{}}{{}{{}}}}, etc., which represent HF-sets in the evident way, this coding being denoted as β: $\{"\{", "\}"\}^* \to HF$, 2) finite trees which are graphs of the special kind and are given by 0-1-incidence matrices; the coding is denoted as τ:Trees $\to$ HF and is defined inductively by τ(tree) = $\{\tau(\text{subtree}1),\ldots,\tau(\text{subtree}k)\}$ for all immediate subtrees of any given tree; 3) graph coding or collapsing $\chi(g,v)$ which assigns to any finite acyclic directed graph g with the distinguished vertex v, a set $\chi(g,v) \in HF$ by the same as τ does.

Example of P^* & $\neg$P-coding is arithmetical one $e:\omega \to HF$ [1] where $e(n) = \{e(n_1),\ldots,e(n_k)\}$ for $n = 2^{n_1} + \ldots + 2^{n_k}$, $n_1 > n_2 > \ldots > n_k$. More exactly, we should distinguish between unary, binary and in general m-ary arithmetical coding e_1, e_2, and e_m, $m \geq 1$, relative to chosen representation of natural numbers by a set of digits $0,1,\ldots,m-1$, i.e.,by m-adic numeric system.

PROPOSITION 1. (1) $\vartheta \leq_P \chi$ holds for any P^*-coding ϑ. (2) The following not invertible P-reducibilities hold: $e_1 \leq_P e_m \equiv_P e_n \leq_P \beta \equiv_P \tau \leq_P \chi$ for $m,n \geq 2$.

PROPOSITION 2. P_χ not $\subseteq P_\beta$; P_β not $\subseteq P_\chi$; $P_\chi \cap P_\beta$ not $\subseteq P_{e_m}$; P_{e_m} not $\subseteq P_\chi, P_\beta$; P_{e_2} not $\subseteq P_{e_1}$; P_{e_1} not $\subseteq P_{e_2}$.

Define set-theoretic Δ-language (cf. [6-8]) consisting of Δ-terms a,b,... and Δ-formulas $\varphi,\psi,\ldots$ by the clauses:

Δ-formulas::= $a \in b \mid \neg\varphi \mid \varphi \& \psi \mid \varphi \vee \psi \mid \varphi \Rightarrow \psi \mid \forall x \in a\varphi \mid \exists x \in a\varphi$;

Δ-terms::= <set-variables> $\mid \{a,b\} \mid \cup \{b(x): x \in a \;\&\; \varphi(x)\} \mid$
$[p = p \cup \{x \in a:\varphi(x,p)\}]$.

Here (closed) variables x and p are different and not occurring in a. These selfexplanatory constructs have the evident semantics in HF (and even in any universe V for ZF) and are every-day used tools of the "working mathematician". The only construct which deserves special definition is inductive Δ-separation $[p = p \cup \{x \in a:\varphi(x,p)\}]$ (its omission essentially gives rise to Kripke-Platek theory without foundation axiom; cf. [1, 6,7]). It is considered as the term (not formula!) and denotes the distinguished solution p of the equation in square brackets obtained as the result of stabilization (in $\leq$ card(a) steps) of monotone sequence $\emptyset = p_0 \subseteq p_1 \subseteq \ldots \subseteq a$, where $p_{n+1} := p_n \cup \{x \in a:\varphi(x,p_n)\}$. Let us call Δ-operations those definable by Δ-terms.

THEOREM 1 (V.Yu.Sazonov). Given any finite alphabet A, there exists the retraction pair $i:HF \to A^*$ and $i^R:A^* \to HF$ ($ii^R = $ identity$:A^* \to A^*$) such that arbitrary P-function $f:A^* \to A^*$ satisfy $fi = i\tilde{f}$ for some Δ-operation $\tilde{f}:HF \to HF$.

Let $\vartheta^R:HF \to A^*$ be a retraction of any coding ϑ. Denote $\Delta_{\tilde{\vartheta}} :=$ Δ-language extended by the corresponding retraction pair $\tilde{\vartheta} := \vartheta i$ and $\tilde{\vartheta}^R := i^R \vartheta^R:HF \to HF$. Any P-coding ϑ with a retraction ϑ^R is called P-regular if the following three functions $A^* \to A^*$ are in P: (1) $\vartheta^R i^R$, which transforms any code a in A^* to the code of a (i.e. of its representation in HF), (2) $i\vartheta$, , which restore the code a in A^* from the code of a, and (3) $\vartheta^R\vartheta$, which transforms any code in A^* to some equivalent code called canonical one.

PROPOSITION 3. Graph, tree and bracket P-coding χ, τ, and β are P-regular, however arithmetical codings (which are only P^*-codings) are not.

THEOREM 2. $\Delta_\chi \equiv P_\chi$, i.e. Δ_χ-language represents exactly P_χ-operations (and P_χ-predicates) in HF (cf. [6-8]). Analogously, $\Delta_\beta \equiv P_\beta \equiv \Delta_\tau \equiv P_\tau$

THEOREM 3 (V.Yu.Sazonov). In general, for any P-regular coding ϑ, η:

(1) $\Delta_\vartheta \equiv P_\vartheta$;

(2) $\vartheta \leq_P \eta$ & $\vartheta \not\geq_P \eta$ implies P_ϑ not $\subseteq P_\eta$ and P_η not $\subseteq P_\vartheta$ (with contraexamples $\tilde{\vartheta}^R$ and $\tilde{\eta}$, respectively);

(3) in fact, $\eta \leq_P \vartheta \iff \tilde{\eta} \in P_\vartheta \iff \tilde{\vartheta}^R \in P_\eta$.

REFERENCES

1. BARWISE J.K. Admissible Sets and Structures, Springer, Berlin, 1975.

2. DAHLHAUS E. Is SETL a suitable language for parallel programming - a theoretical approach. CSL 87, LNCS 329, E.Borger, H.Kleine Buning M.M.Richter editors,pp. 56-63.

3. ERSHOV Yu.L. Numbering theory (In Russian), izd."Nauka", glavnaja red. fiz.-mat. literatury, M., 1977.

4. GANDY R.O. Set-Theoretic Functions for Elementary Syntax, Proceedings in Pure Mathematics, Vol. 13, Part II, 1975, pp. 103-126.

5. SAZONOV V.Yu. Polynomial computability and recursivity in finite domains. Elektronische Informationsverarbeitung und Kybernetik, Vol. 16, N7, 1980 , p. 319-323.

6. SAZONOV V.Yu. Bounded set theory and polynomial computability. All Union Conference on Applied Logic, Proc.,Novosibirsk,1985, p.188-191 (In Russian).

7. SAZONOV V.Yu. Bounded set theory, polynomial computability and Δ-programming. Application aspects of mathematical logic. Computing systems, Vol.122, 1987, p.110-132 (In Russian). See also the short English version of this paper in Lecture Notes in Computer Science, N 278, 1987, p. 391-397.

8. SAZONOV V.Yu. Hereditarily-finite sets with attributes, data bases and polynomial-time computability, Actes Preliminaires, du Symposium Franco-Sovietique, INFORMATIKA'91, 16-18 October, 1991, Grenoble, INRIA, 1991, p.113-132.